

UPMO2016 QUESTIONS

- (1) Evaluate the integral

$$\int \left(1 + x - \frac{1}{x}\right) e^{\left(x + \frac{1}{x}\right)} dx.$$

- (2) Solve the system of equations

$$\begin{cases} x + [y] + \{z\} = 1.1 \\ [x] + \{y\} + z = 2.2 \\ \{x\} + y + \{z\} = 3.3, \end{cases}$$

where  $[a]$  is an integer and  $\{a\} = a - [a]$  is a fractional parts of  $a \in \mathbb{R}$ , respectively.

- (3) Let  $a$  and  $b$  be elements in a ring such that  $(a + b)^n = a^n + b^n$  for  $n = 2$  and  $3$ . Show that  $(a + b)^n = a^n + b^n$  holds for any natural  $n$ .
- (4) Draw the graph of the function

$$y = f(x) = \lim_{n \rightarrow \infty} \left( \cos \frac{x}{\sqrt{n}} \right)^n.$$

- (5) Find the value of the function

$$y = f(x) = \lim_{n \rightarrow \infty} \left( \cos \frac{x}{\sqrt{n}} \right)^n$$

at  $x_0 = \sqrt{2}$ .

- (6) Find  $f^{(2016)}(0)$  if  $f(x) = \sin(x^2)$ .

- (7) Let  $f(x) = \sqrt{x}$  and  $g(x) = Ax + B$ . Find such  $A$  and  $B$  that  $\int_0^1 (f(x) - g(x))^2 dx$  takes the minimal value.

- (8) Let  $x, y$  and  $z$  be positive real numbers less than 4. Prove that at least one of the numbers  $\frac{1}{x} + \frac{1}{4-y}$ ,  $\frac{1}{y} + \frac{1}{4-z}$ ,  $\frac{1}{z} + \frac{1}{4-x}$  is not less than 1.

- (9) What is the maximal value of  $|z|$  if the complex number  $z$  satisfies the condition  $\left|z + \frac{1}{z}\right| = 1$ .

- (10) Find the order of the determinant that the equation

$$\begin{vmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 2 & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & x \end{vmatrix} = 2016$$

has a root at  $x = 6$ .