

UPMO2016 QUESTIONS

- (1) Evaluate the integral

$$\int \left(1 + x - \frac{1}{x}\right) e^{(x+\frac{1}{x})} dx.$$

- (2) Solve the system of equations

$$\begin{cases} x + [y] + \{z\} = 1.1 \\ [x] + \{y\} + z = 2.2 \\ \{x\} + y + \{z\} = 3.3, \end{cases}$$

where $[a]$ is an integer and $\{a\} = a - [a]$ is a fractional parts of $a \in \mathbb{R}$, respectively.

- (3) Let a and b be elements in a ring such that $(a+b)^n = a^n + b^n$ for $n = 2$ and 3 .

Show that $(a+b)^n = a^n + b^n$ holds for any natural n .

- (4) Draw the graph of the function

$$y = f(x) = \lim_{n \rightarrow \infty} \left(\cos \frac{x}{\sqrt{n}} \right)^n.$$

- (5) Find the value of the function

$$y = f(x) = \lim_{n \rightarrow \infty} \left(\cos \frac{x}{\sqrt{n}} \right)^n$$

at $x_0 = \sqrt{2}$.

- (6) Find $f^{(2016)}(0)$ if $f(x) = \sin(x^2)$.

- (7) Let $f(x) = \sqrt{x}$ and $g(x) = Ax + B$. Find such A and B that $\int_0^1 (f(x) - g(x))^2 dx$ takes the minimal value.

- (8) Let x, y and z be positive real numbers less than 4. Prove that at least one of the numbers $\frac{1}{x} + \frac{1}{4-y}$, $\frac{1}{y} + \frac{1}{4-z}$, $\frac{1}{z} + \frac{1}{4-x}$ is not less than 1.

- (9) What is the maximal value of $|z|$ if the complex number z satisfies the condition $|z + \frac{1}{z}| = 1$.

- (10) Find the order of the determinant that the equation

$$\begin{vmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 2 & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & x \end{vmatrix} = 2016$$

has a root at $x = 6$.